International Conference on Boundary and Interior Layers (BAIL): Asymptotic and Numerical Methods

18 – 22 June, 2018

Contents

Introduction

Dear Participant,

On behalf of the University of Strathclyde's Numerical Analysis and Scientific Computing Group, it is our pleasure to welcome you to the *International Conference in Boundary and Interior Layers: Computational and Asymptotic Methods, BAIL 2018.* This follows the long tradition of BAIL meetings starting 38 years ago in Dublin, and it is our great pleasure that Glasgow has been chosen as the city to host the first one of this series to take place in the UK. We are looking forward to meeting delegates from around the world.

We have been very lucky to secure a stellar list of invited speakers who will address us on a variety of topics linked to the numerical solution of problems presenting layers. So, hopefully there is something to suit everyone's interests. Although the meeting is funded almost entirely from the registration fees of the participants, additional financial support has been provided by the *Edinburgh Mathematical Society*, and the *Glasgow Mathematical Journal Trust*.

As well as attending the scientific sessions, we hope you will also take advantage of the opportunity to renew old friendships and meet new people as part of the communal meals and social programme. We are indebted to the *Glasgow City Council* for generously sponsoring a wine reception at the City Chambers on Monday evening. The building is well worth a visit, featuring a vast range of ornate decoration complete with the largest marble staircase in Western Europe. On Wednesday afternoon we will have an excursion, with a visit to the Glengoyne distillery (a 45 minute drive outside Glasgow), and a walk around Luss (a village next to Loch Lomond), or an alternative plan if the weather does not want it that way. The conference dinner on Thursday night will be held in another one of Glasgow's historic buildings, the Lighthouse. This building, linked to Charles Rennie Mackintosh, is a landmark of Glasgow's architecture, and you will hear many details about it during the tour that will take place just before dinner is served.

Thank you for coming, and we hope you enjoy the meeting!

Gabriel R. Barrenechea
Philip Knight
Conference Organising Committee

Information for participants

- **General**. There will be a registration desk in the foyer of the Stenhouse Wing. This is building 14 on the campus map, and has two entrances, one from Cathedral Street, and the other one from the Sculpture Gardens. The organisers can be contacted there during tea and coffee breaks.
- **Lecture room**. All talks will take place on SW 105, in the Stenhouse Wing.
- **Accommodation**. All rooms are in the Campus Village. Check-out time is 10:00 on day of departure. On Friday morning, luggage may be left in the room SW 105.
- **Meals**. Breakfast (Mon-Fri) is available from 08:00 until 09:30 in the TIC building (building 37 in the campus map). The times of lunches and dinners are as indicated in the conference programme. Dinner (Mon-Wed) will also be served in the TIC building. Lunches are served in the TIC building, with the exception of Wednesday's and Friday's, that will be served in the Stenhouse foyer. Coffee and tea will be provided at the advertised times in the foyer outside SW 105.
- **Chairing sessions**. It is hoped that if you are listed as chairing a session, you will be willing to help in this way. Please keep speakers to the timetable!
- **Reception**. A reception for all participants hosted by Glasgow City Council will be held in the City Chambers on Monday 18th June from 19.30 to 21.00.
- **Excursion**. The day out will start on Wednesday at 13:30, when the bus will leave from the Livingstone Tower (26, Richmond Street, building 23 on the campus map). The scheduled return time is 19:00h.
- **Conference dinner**. The conference dinner will be held in the Lighthouse on Thursday 21st June at 18:00 (for 19:00 dinner). There will be a guided tour of the Lighthouse at 18:00, and it will last for about an hour. The venue is located at 11 Mitchell Lane, Glasgow G1 3NU, which is 10 minutes away from the conference venue.
- **Internet access**. Complimentary WiFi is available throughout the campus from the 'WifiGuest' network, which should appear on the list of available networks on your portable device. This network uses the same authentication system as 'The Cloud' network found in public places across the UK. You can log in to 'WifiGuest' using existing 'The Cloud' credentials, or set up a new account which you can then use wherever 'The Cloud' is available. More comprehensive internet access is available to eduroam users.
- **Sports facilities**. Conference delegates can use the University sports facilities (building 1) by obtaining a card from the Student Village Office. The cost of the facilities varies.

Invited Speakers

Víctor Calo	Curtin University, Australia	victor.calo@curtin.edu.au
Alexandre Ern	ENPC, Paris	alexandre.ern@enpc.fr
Emmanuil Georgoulis	Leicester, UK, and Athens, Greece	emmanuil.georgoulis@le.ac.uk
Volker John	WIAS, and Free University, Berlin, Germany	john@wias-berlin.de
Frédéric Valentin	LNCC, Petrópolis, Brazil	valentin@lncc.br

Abstracts of Invited Talks

Stabilization of convection-dominated diffusion with optimal test functions

Víctor M Calo,[1,2] N Collier,[3] A Niemi,[4] A Romkes,[5] E Valseth[5]
[1] Curtin University, Australia
[2] Commonwealth Scientific and Industrial Research Organisation (CSIRO), Australia
[3] Oak Ridge National Laboratory, USA
[4] University of Oulu, Finland
[5] South Dakota School of Mines & Technology, USA

We discuss the class of discontinuous Petrov-Galerkin (DPG) methods for finite element (FE) approximations of boundary value problems for singularly-perturbed, second-order linear partial differential equations (PDEs). We will first introduce the DPG method with optimal basis functions as described by Demkowicz and Gopalakrishnan [1,2] and adapted for advection-diffusion problems by Niemi et al. [3-5]. We will then present a new DPG method which uses a first-order system weak representation of the governing PDEs. This new hybrid continuous-discontinuous DPG method uses classical continuous function spaces for the trial functions, to reduce computational cost and discontinuous function spaces for the test functions. The broken weighting spaces allow us to solve the optimal test functions locally at the element level by using the DPG philosophy. These weighting functions are optimal in the sense that they guarantee inherently stable FE approximations with best approximation properties in the energy norm. We compute the local test-function contributions numerically on each element with high accuracy without solving global variational statements. We will use 2D numerical verifications and convergence studies to validate our analysis. In particular, we will focus on convection-dominated diffusion problems with highly oscillatory (diffusion) coefficients.

References

[1.] L Demkowicz, J Gopalakrishnan, "A class of discontinuous Petrov-Galerkin methods. Part I: The transport equation," Computer Methods in Applied Mechanics and Engineering 199 (23-24), 1558-1572, 2010
[2.] L Demkowicz, J Gopalakrishnan, "A class of discontinuous Petrov-Galerkin methods. II. Optimal test functions," Numerical Methods for Partial Differential Equations 27 (1), 70-105, 2011
[3.] AH Niemi, NO Collier, VM Calo, "Discontinuous Petrov-Galerkin method based on the optimal test space norm for one-dimensional transport problems," Proc Computer Science 4:1862-1869, 2011
[4.] AH Niemi, NO Collier, VM Calo, "Automatically stable discontinuous Petrov-Galerkin methods for stationary transport problems: Quasi-optimal test space norm," Computers & Mathematics with Applications 66 (10), 2096-2113, 2013
[5.] AH Niemi, NO Collier, VM Calo, "Discontinuous Petrov-Galerkin method based on the optimal test space norm for steady transport problems in one space dimension," Journal of Computational Science 4 (3), 157-163, 2013

Hybrid High-Order methods: overview and recent advances

Alexandre Ern
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Hybrid High-Order (HHO) methods have been recently introduced for diffusion problems in [D. Di Pietro, A. Ern, and S. Lemaire, Comp. Methods Appl. Math., 14(4):461-472, 2014] and for linear elasticity problems in [D. Di Pietro and A. Ern, Comp. Meth. Appl. Mech. Eng., 283:1-21, 2015]. HHO methods are formulated in terms of face unknowns which are polynomials of some degree $k \geq 0$. Cell unknowns are also introduced in the formulation and can be eliminated locally by a static condensation procedure. HHO methods are devised from a local reconstruction operator and a local stabilization operator in each mesh cell. This leads to a discretization method that delivers, for smooth solutions and on general meshes with polyhedral cells and non-matching interfaces, error estimates of order $(k+1)$ in the H^1-seminorm (and of order $(k+2)$ in the L^2-norm under full elliptic regularity). Positioning unknowns at mesh faces is also a natural way to express local balance properties. HHO methods have been bridged to Hybridizable Discontinuous Galerkin (HDG) methods in [B. Cockburn, D. Di Pietro, and A. Ern, ESAIM Math. Mod. Numer. Anal., 50(3):635-650, 2016]. In this talk, we review the devising and properties of HHO methods. Then, we show how the method can be used to approximate in a robust manner convection-diffusion problems and, finally, we review recent developments on the use of unfitted meshes to simulate fictitious domain and interface problems.

Interface Layers

Emmanuil H. Georgoulis
University of Leicester, UK and National Technical University of Athens, Greece

The study of interface problems is abundant in mathematical modelling and engineering applications. A particular class of interface problems, motivated by models of mass transfer of solutes through semi-permeable membranes and related process is discussed. More specifically, a model problem consisting of a system of semilinear parabolic advection-diffusion-reaction partial differential equations in each compartment, equipped with respective initial and boundary conditions, is considered. Nonlinear interface conditions modelling selective permeability, congestion and partial reflection are applied to the compartment interfaces leading to possibly the presence of steep gradients at the vicinity of the interfaces. An interior penalty dG method is presented for this problem and it is analysed in the space-discrete setting. The a priori analysis shows that the method yields optimal a priori bounds, provided the exact solution is sufficiently smooth. Further, a posteriori error analysis for the challenging case of curved interfaces is also presented by generalising the dG method presented to meshes comprising of general-shaped elements. Numerical experiments indicate agreement with the theoretical bounds and highlight the stability of the numerical method in the advection-dominated regime.

Finite Elements for Scalar Convection-Dominated Equations and Incompressible Flow Problems: a Never Ending Story?

Volker John & Julia Novo & Petr Knobloch

Weierstrass Institute for Applied Analysis and Stochastics, Mohrenstr. 39, 10117 Berlin, Germany, and Freie Universität Berlin, Department of Mathematics and Computer Science, Arnimallee 6, 14195 Berlin, Germany

The contents of this talk is twofold. First, important recent results concerning finite element methods for convection-dominated problems and incompressible flow problems are described that illustrate the activities in these topics. Second, a number of, in our opinion, important open problems in these fields are discussed. The exposition concentrates on H^1-conforming finite elements.

MHM Methods for Fluids

Frédéric Valentin
LNCC - National Laboratory for Scientific Computing Av. Getulio Vargas, 333 - 25651-075 Petrópolis - RJ, Brazil, and INRIA Sophia Antipolis Méditerranée 2004 Route des Lucioles, 06902 Valbonne, France

This work presents a family of finite element methods for multiscale fluid problems, named Multiscale Hybrid-Mixed (MHM) methods. The MHM method is a consequence of a hybridization procedure at the continuous level which characterizes the unknowns as a direct sum of a “coarse” solution and the solutions to problems with Neumann boundary conditions driven by the multipliers. As a result, the MHM method becomes a strategy that naturally incorporates multiple scales through multiscale basis functions while providing solutions with high-order precision for the primal and dual variables. The completely independent local problems are embedded in the upscaling procedure, and then, computational approximations may be naturally obtained in a parallel computing environment. The numerical analysis for the one- and two-level versions of the MHM method shows that they are optimally convergent and achieve super-convergence for the locally conservative velocity field on general meshes. An underlying multiscale face-based a posteriori estimator is proposed which is locally efficient and reliable in the natural norms. The general framework and recent results are illustrated for the Darcy, Stokes and Brinkman equations, and validated through a large variety of numerical results for highly heterogeneous coefficient problems.

References :

[1] C. Harder, D. Paredes and F. Valentin *A family of multiscale hybrid-mixed finite element methods for the Darcy equation with rough coefficients.* J. Comput. Phys., Vol. 245, pp. 107-130, 2013.
[2] R. Araya, C. Harder, D. Paredes and F. Valentin *Multiscale hybrid-mixed method.* SIAM J. Numer. Anal., Vol. 51, No. 6, pp. 3505-3531, 2013.
[3] C. Harder, D. Paredes and F. Valentin. *On a multiscale hybrid-mixed method for advective-reactive dominated problems with heterogenous coefficients.* SIAM Multiscale Model. and Simul., Vol. 13, No. 2, pp. 491–518, 2015.

[4] C. Harder, A. L. Madureira and F. Valentin. *A hybrid-mixed method for elasticity.* ESAIM: Math. Model. Num. Anal., Vol. 50, No. 2, pp. 311–336, 2016.
[5] C. Harder and F. Valentin. *Foundations of the MHM method.* G. R. Barrenechea, F. Brezzi, A. Cangiani, E. H. Georgoulis Eds. Building Bridges: Connections and Challenges in Modern Approaches to Numerical Partial Differential Equations, Springer, Lecture Notes in Computational Science and Engineering, 2016
[6] D. Paredes, F. Valentin, and H. M. Versieux. *On the robustness of multiscale hybrid-mixed methods.* Math. Comp., Vol. 86, No. 304, pp. 525–548, 2017.
[7] R. Araya, C. Harder, A. Poza and F. Valentin. *Multiscale Hybrid-Mixed Method for the Stokes and Brinkman Equations – The Method.* Comp. Meth. Appl. Mech. Eng., Vol. 324, pp. 29–53, 2017.

Minisymposia abstracts

Minisymposium M1

Recent advances in finite element methods on anisotropic meshes

Organiser
Natalia Kopteva

A unified framework for time-dependent singularly perturbed problems with discontinuous Galerkin methods in time

Gunar Matthies & Sebastian Franz *(Technische Universität Dresden)*

We present a general numerical analysis for a wide class of time-dependent singularly perturbed problems which are discretised in time by discontinuous Galerkin methods. Our analysis is applicable to numerous different spatial operators.
We consider for a bounded Lipschitz domain $\Omega \subset \mathbb{R}^d$, $d \geq 1$, and a final time $T > 0$ the problem

$$\begin{cases} \partial_t u + Lu = f & \text{in } \Omega \times (0,T), \\ Bu = 0 & \text{on } \partial\Omega \times (0,T), \\ u(0) = u_0 & \text{in } \Omega, \end{cases}$$

where L is a possibly singularly perturbed differential operator in space, B a boundary operator, and u_0 an initial value in Ω.
Examples for such problems are

$$Lu = -\varepsilon \triangle u + b \cdot \nabla u + cu, \quad Bu = u$$

as a second-order elliptic convection-diffusion problem with homogeneous Dirichlet conditions or

$$Lu = \varepsilon^2 \triangle^2 u + b \triangle u + cu, \quad Bu = (u, \partial_n u)^T$$

as a fourth-order problem with homogeneous Dirichlet and Neumann conditions.
Looking at the second-order elliptic convection-diffusion problem, a standard Galerkin FEM with Q_p-elements on a two-dimensional Bakhvalov–Shishkin mesh with N cells in each dimension for the spatial approximation and a temporal time discretisation with dG(q) lead to a convergence rates

$$\|u - U\|_\infty + \gamma \|u - U\|_Q \leq C(\tau^{q+1} + N^{-p})$$

in the case of sufficiently small ε where

$$\|v\|_\infty = \sup_{t \in [0,T]} \|v(t)\|_{L^2} \quad \text{and} \quad \|v\|_Q^2 = \sum_{n=1}^{N} Q_n \left[\|v\|_{1,\varepsilon}^2 \right]$$

with the ε-weighted H^1-norm and the Gauss-Radau quadrature Q_n on the n-th time interval.
It is known that dG time discretisations for non-singularly perturbed problems provide superconvergence results at the endpoints of the time intervals. Thus, we also check the error

$$\|u - U\|_{\infty,d} \leq C(\tau^{2q+1} + N^{-p})$$

in the discrete maximum norm in time.
All computations were performed in $\mathbb{SOFE}$, a Matlab/octave-FEM suite initiated by Lars Ludwig.

References :

[1] S. Franz and G. Matthies *A Unified Framework for Time-Dependent Singularly Perturbed Problems with Discontinuous Galerkin Methods in Time*, Math. Comp, 2018, DOI: https://doi.org/10.1090/mcom/3326

Supercloseness of edge stabilization on Shishkin rectangular meshes for convection–diffusion problems with exponential layers

Martin Stynes & Xiaowei Liu & Jin Zhang *(Beijing Computational Science Research Center)*

A finite element continuous interior penalty (CIP) method is applied on a Shishkin mesh to solve a singularly perturbed convection-diffusion problem posed on the unit square, whose solution exhibits exponential and corner layers. This method is closely related to one considered in Franz et al. (J. Comput. Math. 2010), where it is proved that $\|\pi u - u^N\|_{CIP} \leq C(\varepsilon^{1/2}N^{-1} + N^{-3/2})$, where ε is the small diffusion coefficient, πu is a certain projection of the solution u into the finite element space (which comprises piecewise bilinears and piecewise linears on a hybrid rectangular/triangular mesh), N is the number of mesh intervals in each coordinate direction, u^N is the computed solution, and $\|\cdot\|_{CIP}$ is a norm that is naturally suited to the method. For our method, this bound will be improved here to $\|u^I - u^N\|_{CIP} \leq C(\varepsilon N^{-3/2} + N^{-7/4})$, where u^I is the piecewise bilinear nodal interpolant of u on a rectangular mesh. This supercloseness result enables a simple postprocessing of u^N yielding an improved approximant of u. Furthermore, it will be shown in one dimension that, on a uniform mesh with N subintervals, the inequality $\|w\|_{SD} \leq CN^{1/4}\|w\|_{CIP}$ for all piecewise linear w is sharp (here $\|\cdot\|_{SD}$ is the streamline diffusion norm), which clarifies the hitherto imprecise relationship between these two norms.

On the inf-sup stability of the lowest order Taylor-Hood pair on anisotropic meshes

Andreas Wachtel *(ITAM, México)* & G. R. Barrenechea *(University of Strathclyde)*

Uniform LBB conditions are desirable to approximate the solution of Navier-Stokes, Oseen, and Stokes equations on anisotropic meshes and to enable anisotropic refinements. This talk will be devoted to present the first proof (to our best knowledge) of the uniform stability the Taylor–Hood pairs $\mathbb{Q}_2 \times \mathbb{Q}_1$ and $\mathbb{P}_2 \times \mathbb{P}_1$ on a class of anisotropic meshes. These meshes may contain refined edge and corner patches. To prove this, we generalised Verfürth's trick and recent results by the authors.
Previous results [1,2] were mainly negative, in the sense that there were meshes for which the inf-sup constants degenerate with the aspect ratio. We have then proposed minimal conditions for the lowest order Taylor Hood pair to be uniformly stable with a constant independent of the aspect ratio.
The main ideas of the proof are presented, and also we show some possible extensions and numerical evidence.

References :

[1] T. Apel and H. Maharavo Randrianarivony, *Stability of discretizations of the Stokes problem on anisotropic meshes*, Math. Comput. Simulation, 61 (2003), pp. 437–447.
[2] D. Schötzau, C. Schwab, and R. Stenberg, *Mixed hp-FEM on anisotropic meshes. II. Hanging nodes and tensor products of boundary layer meshes*, Numer. Math., 83 (1999), pp. 667–697.

Supercloseness of continuous interior penalty methods on Shishkin triangular meshes and hybrid meshes

Jin Zhang *(Shandong Normal University)* & Xiaowei Liu *(Qilu University of Technology (Shandong Academy of Sciences))*

In this talk, we will present a new supercloseness result of a continuous interior penalty CIP method on a Shishkin triangular mesh or a Shishkin hybrid mesh consisting of triangles and rectangles. For the CIP method, a variant of Oswald interpolation operator is introduced for a discrete inf-sup stability. This stability and a new cancellation technique enable new supercloseness results for the CIP method: the computed solutions on the triangular mesh and the hybrid mesh are shown to be 3/2 order and 2 order (up to a logarithmic factor) convergent in the new norm to the interpolants of the true solution, respectively. These convergence orders are uniformly valid with respect to the diffusion parameter and imply that for the Shishkin mesh the hybrid mesh is superior to the triangular one.

Minisymposium M2
Positivity-preserving discretisations
Organiser Gabriel R. Barrenechea

On Solving the nonlinear problems arising in algebraic stabilizations of steady-state convection-diffusion equations

Abhinav Jha & Volker John *(WIAS, Berlin)*

Algebraic stabilizations, also called Algebraic Flux Correction (AFC) schemes, belong to the very few finite element discretizations of steady-state convection-diffusion equations that obey the discrete maximum principle. However, appropriate limiters depend on the solution itself, thus leading to a nonlinear discrete problem. The efficient solution of these nonlinear problems seems currently to be the biggest drawback in the application of these schemes.

In this talk, several methods for solving the nonlinear problems are introduced. The focus is particularly on Newton's method, thereby discussing regularizations of the limiters to obtain differentiable expressions. Two limiters will be considered, the traditional Zalesak (or Zalesak–Kuzmin) limiter and a recently proposed limiter that is linearity preserving. Numerical studies assess the different iterative schemes.

A unified analysis of AFC schemes for convection–diffusion equations

Petr Knobloch *(Charles University, Prague)*, Gabriel R. Barrenechea & Volker John

This contribution is devoted to the application of algebraic flux correction (AFC) finite element schemes to the numerical solution of scalar steady-state convection–diffusion equations. In contrast to the most finite element stabilization techniques, which are based on variational formulations, the idea of AFC schemes is to modify the algebraic system corresponding to the discrete problem. The basic philosophy of AFC schemes was formulated already in the 1970s. In the last fifteen years, these methods have been intensively developed by Dmitri Kuzmin and his coworkers, see, e.g., [5,6]. An important feature of AFC schemes is the satisfaction of the discrete maximum principle without a pronounced smearing of the layers.

A rigorous analysis of AFC schemes was missing for a long time. In fact, the first contributions to the numerical analysis of AFC schemes were presented only recently in [2,3,4]. The first paper [2] focuses on the solvability of the nonlinear scheme, while [3] presents the first error analysis of AFC schemes. Interestingly, the paper [3] also presented negative results showing that unless some restrictions are imposed on the mesh, the numerical scheme may not converge. Finally, in the paper [4] the discrete maximum principle was established on general meshes and the role of the linearity preservation was studied. This study is also complemented by the work [1], where a link between AFC schemes and a nonlinear edge-based diffusion scheme is presented, and the linearity preservation of the scheme is also studied in detail.

In the present contribution, a general edge-based AFC scheme is considered and the above-mentioned results are presented in a unified way under very general assumptions. In addition, new results on the discrete maximum principle are established. Specific versions of the general AFC scheme corresponding to various methods published in the literature are reviewed and their main properties are stated. Numerical studies compare the different versions of the scheme.

References

[1] G.R. Barrenechea, E. Burman, F. Karakatsani, Edge-based nonlinear diffusion for finite element approximations of convection–diffusion equations and its relation to algebraic flux-correction schemes. *Numer. Math.* 135 (2017), 521–545.

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Analysis and design of algebraic flux correction schemes for eigenvalue range preserving transport of symmetric tensor fields

Christoph Lohmann & Dmitri Kuzmin *(Institute of Applied Mathematics (LS III), TU Dortmund University)*

This work extends the algebraic flux correction paradigm [2] to finite element discretizations of conservation laws for symmetric tensor fields. The proposed algorithms are designed to enforce discrete maximum principles and preserve the eigenvalue range of evolving tensors. To that end, a continuous Galerkin approximation is modified by adding a linear artificial diffusion operator and a nonlinear antidiffusive correction. The latter is decomposed into edge-based fluxes and constrained to prevent violations of local bounds for the minimal and maximal eigenvalues. In contrast to the flux-corrected transport (FCT) algorithm developed in [3] and geometric slope limiting techniques for stress tensors [4], the admissible eigenvalue range is defined implicitly and the limited antidiffusive terms are incorporated into the residual of the nonlinear system. In addition to scalar limiters that use a common correction factor for all components of a tensor-valued antidiffusive flux, tensor limiters are designed using spectral decompositions. The new limiter functions are analyzed using tensorial extensions of the theoretical framework developed in [1] for scalar convection-diffusion equations. The proposed methodology is backed by rigorous proofs of eigenvalue range preservation and Lipschitz continuity. Convergence of pseudo time-stepping methods to stationary solutions is demonstrated in numerical studies.

References :

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High order algebraic flux correction algorithms based on Bernstein-Bezier finite elements for nonlinear hyperbolic systems

Sibusiso Mabuza, John N. Shadid *(Sandia National Laboratories)*, Dmitri Kuzmin & Christoph Lohmann *(TU Dortmund University)*

An arbitrarily high order algebraic flux correction scheme is proposed for nonlinear hyperbolic systems and highly coupled problems. We make use of the Bernstein-Bezier finite elements to discretize the problem in space. The positivity and partition of unity properties of the Bernstein basis makes them appealing for problems that require positivity preservation. We consider a stabilization approach in which low order diffusion is added to the semi-discrete for of the problem. The result is a semi-discrete form which satisfies the system LED condition. Then element based nodal variational limiters are introduced to make the scheme high resolution. To deal with phase errors, we add background high order dissipation such as SUPG. The stabilized semi-discrete scheme can then be discretized in time using various time integrators. Numerical studies are done to illustrate the performance of the scheme with various scalar problems and with shock hydrodynamics problems.

Abstracts of Contributed Talks

A posteriori error estimates for the Stokes problem with singular sources

Alejandro Allendes, E. Otárola *(Departamento de Matemática, Universidad Técnica Federico Santa María, Chile)* & A. J. Salgado *(Department of Mathematics, University of Tennessee, Knoxville, USA)*

Based on [1], where the authors provide a well-posedness result for the Stokes problem in weighted Sobolev spaces over nonconvex, Lipschitz polytopes, we develop in [2], a posteriori error estimators for classical low–order inf–sup stable and stabilized finite element approximations of the Stokes problem with singular sources in two and three dimensions. The designed error estimators are proven to be reliable and locally efficient. On the basis of these estimators we design a simple adaptive strategy that yields optimal rates of convergence for the numerical examples that we perform.

References :

[1] E. Otárola and A. J. Salgado, *The Poisson and Stokes problems in nonconvex, Lipschitz polytopes*, arXiv:1711.08542, 2017.
[2] A. Allendes, E. Otárola and A. J. Salgado, *A posteriori error estimates for the Stokes problem with singular sources*, preprint, 2018.

Patch-wise local projection stabilized FEM for the Oseen problem

Rahul Biswas *(Indian Institute Of Science)*, Thirupathi Gudi*(Indian Institute Of Science)* & Asha K. Dond *(Indian Institute Of Science)*

In the finite element approximation of the Oseen problem, one needs to handle two main obstacles: the lack of stability due to convection dominance, and incompatibility of the velocity and pressure approximation finite element spaces. These obstacles can be handled with an edge patch-wise local projection (EPLP) stabilization technique. This article analyses the EPLP stabilized nonconforming finite element methods for the Oseen problem. The lowest-order Crouzeix-Raviart (CR) nonconforming finite element space is considered for approximating velocity, whereas for the approximating pressure the piecewise constant space and CR finite element space are considered. The proposed discrete weak formulation is a linear combination of the standard Galerkin method, EPLP stabilization and weakly imposed boundary condition using Nitsche's technique. The resulting bilinear form satisfies the inf-sup condition with respect to EPLP norm, which leads to the well-posedness of the discrete problem. *A priori* error analysis assures the optimal convergence in both approximation cases, that is, order one in piecewise constant approximation to pressure case and 3/2 in CR-finite element approximation.

References :

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Goal-oriented error control for stabilized approximations of convection-dominated problems

Marius Paul Bruchhäuser & Markus Bause *(Helmut Schmidt University, Faculty of Mechanical Engineering, Holstenhofweg 85, 22043 Hamburg, Germany)*

The efficient and reliable approximation of nonstationary convection-diffusion-reaction problems

$$\partial_t u + \boldsymbol{b} \cdot \nabla u - \nabla \cdot (\varepsilon \nabla u) + \alpha u = f \tag{1}$$

with small diffusion $0 < \varepsilon \ll 1$ remains to be a challenging task. The solutions of convection-dominated transport problems are typically characterized by the occurence of sharp moving fronts and interior or boundary layers. For their numerical approximation stabilized methods like the SUPG approach are used. A further and widespread technique to capture singular phenomena and sharp profiles is the application of adaptive mesh refinement based on a posteriori error control mechanism. Even though it seems to be natural to combine stabilized finite element methods with adaptive error control mechanisms to further enhance the approximation quality, this combination has been studied rarely so far in the literature. Existing a posteriori error analyses are either typically based on error norms that are non natural for the stabilized scheme or they are not robust with respect to the small diffusion parameter. Here we combine stabilized finite element methods with an a posteriori error control mechanism based on a dual weighted residual approach [1]. The dual weighted error estimator assesses the discretization error with respect to a given goal quantity of physical interest on a single mesh cell. In contrast to former works on goal-oriented error control for transport problems, we solve the stabilized dual problem by a higher order approach. Avoiding interpolation techniques like in other works is done in order to improve the quality of the numerical approximation and error control in particular in the sensitive regions, where the application of interpolation techniques is expected to be highly defective.
The derivation of our goal-oriented error control for SUPG stabilized approximations of Eq. (1) is presented. Moreover, its numerical performance properties are studied and illustrated for benchmark problems of convection-dominated transport; cf. [3,5,4]. New concepts for the efficient higher order approximation of the dual problem are addressed further; cf. [2].

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A Hybrid High–Order method for flows and passive transport in fractured porous media

Florent Chave & Daniele A. Di Pietro *(University of Montpellier (IMAG), France)* & Luca Formaggia *(Politecnico di Milano (MOX), Italy)*

We consider a hybrid dimensional model for the simulation of Darcy flows in fractured porous media, in which the fracture is considered as an hyperplan that cross our domain of interest. Coupling conditions are derived using an averaging process, that relate jumps and averages of normal component of the bulk flux and bulk pressure with the averaged fracture pressure, and allow discontinuity of unknowns across the fracture (see [1] for further informations about the derivation). Then, we propose a new hybrid dimensional model for the simulation of passive transport in fractured porous media, where the novel coupling equations mimic at the discrete level the property that the advection terms do not contribute to the energy balance. This choice enables us to handle the case where the concentration of the solute jumps across the fracture.
Concerning the space discretization, we focus here on the Hybrid High–Order method, that has several assets (i) it supports fairly general meshes possibly containing polygonal elements and nonmatching interfaces; (ii) it allows arbitrary approximation orders; (iii) it is locally conservative; (iv) the robustness with respect to the heterogeneity and anisotropy of the permeability coefficients. For the version of the method corresponding to

a polynomial degree $k \geq 0$, we prove in [2] convergence in h^{k+1} of the discretization error measured in an energy-like norm.

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Numerical simulation on a fixed mesh of a fluid-structure interaction system with a structure given by a finite number of parameters

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We are dealing with the numerical simulation of a 2D fluid–structure interaction problem. The fluid is represented by the incompressible Navier–Stokes equations and the deformable structure depends on a finite number of parameters. The interaction model is original and has been studied in Delay (2018).
The numerical simulations are realized with a fictitious domain method based on a finite element approximation (CutFEM), see Fournié and al. (2017). The considered mesh is fixed and covers the fluid and the structure domains. The structure is localized by a level–set method.
In the present work, we consider two parameters that give the position of the interface between the fluid and the structure. This interface is approached by a set of points used to update the level–set. This crucial point will be detailed in the presentation. The time evolution is computed with a partitioned approach, the structure is updated first and then the state of the fluid is computed.
This work is a part of a wider study whose goal is to stabilize the fluid around a reference flow, see Delay (2018). In that context, the interface between the fluid and the structure plays a significant role.

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hp-version discontinuous Galerkin methods on polygonal and polyhedral meshes

Zhaonan Dong & Andrea Cangiani & Emmanuil Georgoulis *(Department of Mathematics, University of Leicester)*

In this talk, we will first review the recently developed *hp*-version symmetric interior penalty discontinuous Galerkin (dG) finite element method for the numerical approximation of PDEs on general computational meshes consisting of polygonal/polyhedral (polytopic) elements. The key feature of the proposed dG method is that the stability and *hp*-version a-priori error bound are derived based on the specific choice of the interior penalty parameters which allows for edges/faces degeneration. Moreover, under certain practical mesh assumptions, the proposed dG method was proven to be available to incorporate very general polygonal/polyhedral elements with an *arbitrary* number of faces. Because of utilising general shaped elements, the dG method shows a great flexibility in designing an adaptive algorithm by refining or coarsening general polytopic elements. Especially for solving the convection-dominated problems on which boundary and interior layers may appear and need a lot of degrees of freedom to resolve. In the second part of this talk, we will show the a-posteriori error analysis for the dG method. A series of numerical experiments are presented to demonstrate the performance of the proposed dG method on general polygonal/polyhedral meshes.

Optimizing reduced basis methods for cost-effective computational aerodynamics

Marco Fossati & Gaetano Pascarella *(University of Strathclyde)*

This talk will address the optimization of Reduced Basis Methods based on Proper Orthogonal Decomposition and their application to a series of applied aerodynamic problems over three-dimensional geometries. Centroidal Voronoi tessellation, leave-one-out cross validation, proper orthogonal decomposition, and multidimensional interpolation are integrated to define a reduced-order modeling approach for the parametric evaluation of steady aerodynamic loads. The proper orthogonal decomposition-based methodology allows reducing the number of degrees of freedom of the problem while maintaining good accuracy for the solution of complex three-dimensional viscous turbulent flows. As a result, it yields fairly accurate solutions at a fraction of the time required by standard computational fluid dynamics approaches. The focus will be on the ability to perform cost-effective analyses (steady and time-varying) for aerodynamic design. Interest will be on transonic regimes and multiphysics applications.

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A projection method with grad-div stabilization for the time-dependent Navier-Stokes equations

Bosco García-Archilla *(Universidad de Sevilla)* & Javier de Frutos *(Universidad de Valladolid)* & Julia Novo *(Universidad Autónoma de Madrid)*

We study fully discrete approximations to the time-dependent Navier–Stokes equations by means of inf-sup stable mixed finite elements with standard grad-div stabilization and the Euler incremental projection method in time. It is well-known that for high Reynolds numbers, splitting methods like the one we study here are computationally more efficient than coupled methods, and competitive in accuracy. We prove error bounds where the constants do not depend on negative powers of the viscosity. For the velocity, we show that the decay rate of the error in the mesh diameter is k in $L^\infty(0,T,L^2)$ ($(0,T]$ being the time interval where the numerical approximations are obtained) if piecewise polynomials of degree k are used (assuming enough regularity of the solution). For the pressure, the same rate of decay is obtained for errors in the $L^2(0,T,L^2)$ if piecewise polynomials of degree k are used. In the above mentioned norms the errors decay linearly with the time step size. The proofs are based on consistency plus (nonlinear) stability arguments. We present numerical experiments that confirm both the orders of convergence and the independence of the viscosity of our error bounds.

Mixed finite element approximations based on *hp*-adaptive quarter-point meshes for some singular elliptic problems

Sônia M. Gomes & Philippe R. B. Devloo & Omar Durán & Agnaldo M. Farias & Denise de Siqueira *(IMECC-Universidade Estadual de Campinas, SP, Brazil)*

Singularities in the solution of elliptic problems in bounded domains may occur, for instance, due to the presence of re-entrant boundary corners or of abrupt change in boundary conditions. Different techniques have been proposed as a remedy for the poor convergence results of standard finite element methods of such low regularity behavior, as the addition of special functions to the approximation space, or the use of anisotropic and/or adaptive meshes. The proposed method incorporates these aspects in some extent. Precisely, the purpose is to apply the mixed finite element method based on approximations given by quarter point elements in the vicinity of the singularity. The refinement of quarter-point meshes gives curved anisotropic elements, the shape functions inheriting the singular behavior of the geometric map. These singular elements are widely used in typical elasticity formulations for fracture mechanics applications, where a square root singularity is representative for modeling the state of tension around a fracture, with significant accuracy improvement of the numerical solutions. As far as we understand, the use of quarter-point elements in mixed formulations has not been explored in the literature. In such context, the assembly of the required $\mathbf{H}(\mathrm{div})$-conforming approximations shall be described, for h and hp-refinements of the corresponding curved meshes. Numerical results shall be presented, demonstrating a superior effectiveness of the technique based on quarter point geometry for convergence acceleration of approximate singular solutions, when confronted with usual affine finite elements.

Parameter-uniform global accuracy for singularly perturbed parabolic problems with incompatible boundary-initial data

José Luis Gracia *(IUMA and University of Zaragoza, Spain)* & E. O'Riordan *(Dublin City University, Ireland)*

This is a companion talk to the presentation *Singularly perturbed reaction-diffusion problems with incompatible boundary-initial data* by the same authors. Here we focus on the numerical performance of the method. The solutions of some test problems, involving singularly perturbed parabolic equations of reaction-diffusion type with incompatible initial and boundary data, are approximated using a combination of a numerical method and particular analytical functions. The solution u of these test examples is split into the sum of two components $u = s + y$. The discontinuous function s is appropriately designed in order to match the incompatibility between the initial and boundary data, while the function y is the solution of a singularly perturbed problem with a non-smooth inhomogenous term, but which has compatible boundary-initial data. Numerical approximations to this component y are generated from a standard finite difference operator defined on a priori mesh of Shishkin type. For each of the selected test problems, the parameter-uniform global convergence of the numerical approximations is discussed.

Towards a numerical solution of the Hemker problem

Alan F. Hegarty *(University of Limerick, Ireland)* & E. O'Riordan *(Dublin City University, Ireland)*

In this talk we examine a modified version of the Hemker problem (P. W. Hemker, A singularly perturbed model problem for numerical computation, *J. Comp. Appl. Math*, **76**, 1996, 277–285), which we redefine as the following particular problem: Find u such that

$$-\varepsilon\triangle u + u_x = 0, \quad \text{in} \quad \Omega := \{(x,y)|1 < x^2 + y^2 < 4\};$$
$$u = 0, \quad \text{on } \{(x,y)|x^2 + y^2 = 4\}; \qquad u = 1, \quad \text{on } \{(x,y)|x^2 + y^2 = 1\}.$$

The original problem is in an infinite domain around the unit circle and has an exact solution.
The solution of our problem will have a regular boundary layer to the left of the unit circle and internal characteristic layers emanating from the characteristic points $(0, \pm 1)$. Asymptotic information about the nature and scale of the layers in the vicinity of the characteristic points is incorporated into the overall design of the numerical algorithm and in the construction of various piecewise-uniform Shishkin meshes.
Different coordinate transformations are utilized in the region $x < 0$, near $x = 0$ and in the region $x > 0$. The performance of the resulting numerical method is examined across an extensive range of the singular perturbation parameter ε.

Convergent semi-Lagrangian methods for the Monge-Ampère equation: non-convex domains and boundary layers

Max Jensen *(Sussex University)* & Xiaobing Feng *(University of Tennessee)*

In this presentation I will discuss a semi-Lagrangian discretisation of the Monge-Ampère operator on P1 finite element spaces. The wide stencil of the scheme is designed to ensure uniform stability of numerical solutions. Monge-Ampère equations arise for example in the Inverse Reflector Problem where the geometry of a reflecting surface is reconstructed from the illumination pattern on a target screen and the characteristics of the light source.
Monge-Ampère type equations, along with Hamilton-Jacobi-Bellman type equations are two major classes of fully nonlinear second order partial differential equations (PDEs). From the PDE point of view, Monge-Ampère

type equations are well understood. On the other hand, from the numerical point of view, the situation is far from ideal. Very few numerical methods, which can reliably and efficiently approximate viscosity solutions of Monge-Ampère type PDEs on general convex domains. There are two main difficulties which contribute to the situation:
* Firstly, it is well known that the fully nonlinear structure and nonvariational concept of viscosity solutions of the PDEs prevent a direct formulation of any Galerkin-type numerical methods.
* Secondly, the Monge-Ampère operator is not an elliptic operator in generality, instead, it is only elliptic in the set of convex functions and the uniqueness of viscosity solutions only holds in that space. This convexity constraint, imposed on the admissible space, causes a daunting challenge for constructing convergent numerical methods; it indeed screens out any trivial finite difference and finite element analysis because the set of convex finite element functions is not dense in the set of convex functions
* Thirdly, the Bellman formulation allows the study on the Monge-Ampère problem on non-convex domains with non-convex boundary data. At the transition points from classical Dirichlet conditions to viscosity Dirichlet conditions *boundary layers* need to be carefully approximated.
The goal of our work is to develop a new approach for constructing convergent numerical methods for the Monge-Ampère Dirichlet problem, in particular, by focusing on overcoming the second difficulty caused by the convexity constraint. The crux of the approach is to first establish an equivalent (in the viscosity sense) Bellman formulation of the Monge-Ampère equation and then to design monotone numerical methods for the resulting Bellman equation on general triangular grids. An aim in the design of the numerical schemes was to make Howard's algorithm available, which is a globally superlinearly converging semi-smooth Newton solver as this allows us to robustly compute numerical approximations on very fine meshes of non-smooth viscosity solutions. An advantage of the rigorous convergence analysis of the numerical solutions is the comparison principle for the Bellman operator, which extends to non-convex functions. We deviate from the established Barles-Souganidis framework in the treatment of the boundary conditions to address challenges arising from consistency and comparison. The proposed approach also bridges the gap between advances on numerical methods for these two classes of second order fully nonlinear PDEs.

A splitting linearly implicit method for solving time dependent semilinear reaction-diffusion systems

C. Clavero *(Department of Applied Mathematics and IUMA, University of Zaragoza)* & **Juan Carlos Jorge** *(Department of Computational and Mathematical Engineering and ISC, UPNA)*

In this talk we deal with the numerical solution of 1D semilinear parabolic singularly perturbed systems of reaction-diffusion type, which are given by

$$\begin{cases} L_\varepsilon \mathbf{u} \equiv \dfrac{\partial \mathbf{u}}{\partial t}(x,t) - \mathcal{D}_\varepsilon \dfrac{\partial^2 \mathbf{u}}{\partial x^2} + \mathcal{A}(\mathbf{u}) = \mathbf{0},\ (x,t) \in Q \equiv (0,1)\times(0,T], \\ \mathbf{u}(0,t) = \mathbf{g_1}(t),\ \mathbf{u}(1,t) = \mathbf{g_2}(t),\ \forall\ t \in [0,T],\ \mathbf{u}(x,0) = \boldsymbol{\varphi}(x),\ \forall\ x \in (0,1) \end{cases}$$

where $\mathcal{D}_\varepsilon =$ diag $(\varepsilon_1, \varepsilon_2)$, being $\mathbf{u} = (u_1, u_2)^T$ (initially, we consider only systems with two components to highlight the main qualities of the method in the simplest way), $0 < \varepsilon_1 \le \varepsilon_2 \le 1$, and the reaction term is given by $\mathcal{A}(\mathbf{u}) = (a_1(x,t,\mathbf{u}), a_2(x,t,\mathbf{u}))^T$, with a_i sufficiently smooth functions such that, for all $\mathbf{v} \in \mathcal{R}^2$,

$$\frac{\partial a_i}{\partial v_i}(x,t,\mathbf{v}) > 0,\ \frac{\partial a_i}{\partial v_j}(x,t,\mathbf{v}) \le 0,\ i \ne j,\ i,j = 1,2,$$
$$\min_{-\infty \le \mathbf{u} \le \infty} \left(\frac{\partial a_i}{\partial v_1}(x,t,\mathbf{v}) + \frac{\partial a_i}{\partial v_2}(x,t,\mathbf{v}) \right) \ge \alpha > 0,\ i = 1,2.$$

As well, we assume sufficient smoothness and compatibility conditions for the rest of the data in order to the exact solution $\mathbf{u} \in C^{4,2}(\overline{Q})$. We admit that the diffusion parameters ε_i can be very small and also that they can have different order of magnitude; in such case, overlapping boundary layers appear close to the end points of the interval $(0,1)$.
The numerical method which we propose combines a linearized version of the fractional implicit Euler method together with a splitting by components, to discretize in time, and the central finite difference scheme on an appropriate nonuniform mesh, to discretize in space. In this way, only linear systems are involved in the advance in time. More concisely, the use of the splitting by components technique provokes that only tridiagonal linear systems must be solved at each time level of the discretization.

In the analysis of the uniform convergence, we choose a uniform mesh to discretize in time to simplify the presentation; on the other hand, to discretize in space we use a piecewise uniform mesh of Shishkin type, which concentrates the grid points in the boundary layer regions in a suitable way. Then, the fully discrete scheme is uniformly convergent, of first order in time and of almost second order in space.
We show some numerical experiments, obtained for different test problems, which corroborate in practice the robustness and the efficiency of the studied numerical algorithm. Besides, we show that our proposal can be easily extended to systems with more components. In fact, our algorithm becomes more and more efficient, compared to other methods proposed in the literature, as long as the number of components increases.

Identifying basins of attraction of metastable dynamics via Zubov's method

Dante Kalise & Diego Oyarzún *(Imperial College London)*

A classical problem in nonlinear dynamical systems $\dot{x} = f(x)$ is the identification of basins of attraction for stable equilibria. We address this problem via the so-called Zubov's method, which characterizes the basin of attraction as the set $\{x : v(x) < 1\}$, where $v(x)$ the solution of a first order PDE (Zubov's eqn.)

$$\nabla v(x) \cdot f(x) = -h(x)(1 - v(x))\sqrt{1 + ||f(x)||^2}\,.$$

In this talk, following nonlinear control theory approach we review known and recent results concerning the numerical approximation of Zubov's equation by means of semi-Lagrangian schemes, and we discuss applications in the context of metastable dynamics arising in systems biology.

Impinging jet flow and hydraulic jump on a rotating disk

Roger E. Khayat *(Department of Mechanical and Materials Engineering, University of Western Ontario, London, Ontario, Canada N6A 5B9, rkhayat@uwo.ca)*

The free-surface flow formed by a circular jet impinging on a rotating disk is analyzed theoretically. The study explores the effects of rotation and inertia on the thin-film flow. Both boundary-layer height and film thickness are found to diminish with rotation speed. A maximum film thickness develops in the supercritical region, which reflects the competition between the convective and centrifugal effects. Unlike the flow on a stationary disk, an increase in the wall shear stress along the radial direction is predicted, at a rate that strengthens with rotating speed. Our results corroborate well existing measurements (Ozar et al. 2003). The location and height of the hydraulic jump are determined subject to the value of the thickness at the edge of the disk, which is established first for a stationary disk based on the capillary length, and then for a rotating disk using existing analyses and measurements in spin coating. The case of a stationary disk is revisited in an effort to predict the location and height of the jump uniquely. Despite the prolific number of studies since the seminal work of Watson (1964), this remains an outstanding issue in the literature (see, for instance, Duchesne et al. 2014). In this study, the formulated value of the film height at the edge of the disk seems to give excellent agreement against existing measurements (Dressaire et al. 2010) for a jet at moderately high flow rate or low viscosity where the jump structure is well identifiable in reality. The current study (Yunpeng & Khayat 2018) is the latest in a series of works on the formation of hydraulic jump where we previously explored non-Newtonian effects (Zhao & Khayat 2008) and the influence of slip (Khayat 2016), which I will briefly highlight in my talk.

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A posteriori error estimation on anisotropic meshes: lower and upper error bounds

Natalia Kopteva *(University of Limerick)*

We shall start by reviewing residual-type a posteriori error estimates from [2,3] on anisotropic meshes, i.e. we allow mesh elements to have extremely high aspect ratios. The error constants in these estimates are independent of the diameters and the aspect ratios of mesh elements. Note also that, in contrast to some a posteriori error estimates on anisotropic meshes in the literature (see, e.g., [5,6]), our error constants do not involve so-called matching functions (that depend on the unknown error and, in general, may be as large as mesh aspect ratios). To deal with anisotropic elements, a number of technical issues have been addressed in [2,3]. For example, an inspection of standard proofs for shape-regular meshes reveals that one obstacle in extending them to anisotropic meshes lies in the application of a scaled traced theorem when estimating the jump residual terms (this causes the mesh aspect ratios to appear in the estimator). For maximum norm estimates, the analysis also employs sharp bounds on the Green's function from [1]. For the estimation in the energy norm, a special quasi-interpolation operator is constructed on anisotropic meshes, which may be of independent interest [3].
In the second part of the talk, the focus will shift to the efficiency of energy-norm error estimators on anisotropic meshes. It was previously addressed in [5,6] using the standard bubble function approach. However, a numerical example is given in a more recent paper [4] that clearly demonstrates that short-edge jump residual terms in such bounds are not sharp. To remedy this, we shall present a new upper bound for the short-edge jump residual terms of the estimator. A version of this bound for partially structured anisotropic meshes is given in [4]. We shall also discuss a generalization of this bound to more general anisotropic meshes. This new bound for the short-edge jump residual terms implies that for some anisotropic meshes the error estimator constructed in [3] is efficient.

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Maximum-norm a posteriori error estimates for singularly perturbed parabolic equations

Torsten Linß *(FernUniversität in Hagen)* & Natalia Kopteva

Singularly perturbed parabolic equations of reaction-diffusion type are considered. For these equations, we give a posteriori error estimates in the maximum norm.
Semidiscrete and fully discrete versions of the backward Euler and of the Crank-Nicolson methods are considered. For their full discretisations, we employ elliptic reconstructions that are, respectively, piecewise-constant and piecewise-linear in time. Certain bounds for the derivatives of the Green's function of the parabolic operator are employed.

On some experience with high-order, exactly divergence-free FEM for transient incompressible flow problems

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In [1] we presented a unified approach to exactly divergence-free inf-sup stable FEM for the time-dependent incompressible Navier-Stokes problem, covering both H^1- and $H(div)$-conforming methods. Basic features are pressure-robustness, i.e. additional gradient fields $\nabla\psi_h$ in the source term lead to a change $p_h + \psi_h$ of the pressure. This implies that velocity error estimates are not corrupted by large multiples of the best pressure interpolation error. Moreover, the methods are shown to be semi-robust w.r.t. the Reynolds number Re if $u \in L^1(0,T;W^{1,\infty}(\Omega))$, i.e. the error estimates (including the exponential Gronwall factor) do not explicitly on Re.
In this talk, we report on our numerical experience with benchmark problems in 2D and 3D vortex dynamics using high-order FEM including homogeneous, decaying turbulence, see [2,3]. Moreover, we present some first results on attached boundary layer flows. In particular, we will discuss the question of required numerical diffusion.

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Error analysis of non inf-sup stable discretizations of the time-dependent Navier-Stokes equations with local projection stabilization

Julia Novo *(Universidad Autónoma de Madrid)* & Javier de Frutos *(Universidad de Valladolid) & Bosco García-Archilla* (Universidad de Sevilla) & Volker John *(WIAS, Berlin)*

In this talk we consider non inf-sup stable finite element approximations to the evolutionary Navier–Stokes equations. Several local projection stabilization (LPS) methods corresponding to different stabilization terms are analyzed, thereby separately studying the effects of the different stabilization terms. Error estimates are derived in which the constants are independent of inverse powers of the viscosity. For one of the methods, using velocity and pressure finite elements of degree l, it will be proved that the velocity error in $L^\infty(0,T;L^2(\Omega))$ decays with rate $l + 1/2$ in the case that $\nu \leq h$, with ν being the dimensionless viscosity and h the mesh width. In the analysis of another method, it was observed that the convective term can be bounded in an optimal way with the LPS stabilization of the pressure gradient. Fully discrete schemes with both the implicit Euler and the Crank-Nicolson methods are considered and analyzed. Some numerical studies confirm the analytical results.

Singularly perturbed reaction-diffusion problems with incompatible boundary-initial data

Eugene O'Riordan *(Dublin City University, Ireland)* & J.L. Gracia *(IUMA and University of Zaragoza, Spain)*

Global numerical approximations are constructed for the solution of linear singularly perturbed reaction-diffusion parabolic problems of the form

$$\varepsilon(u_t - u_{xx}) + b(x,t)u = f(x,t), \quad (x,t) \in (0,1) \times (0,T]; \qquad b(x,t) > \beta > 0;$$
$$u(0,t) = g_L(t),\ u(1,t) = g_R(t)\ t \geq 0, \quad u(x,0) = \phi(x),\ 0 < x < 1; \quad \phi(0^+) \neq g_L(0);$$

with an incompatibility between the initial condition and the boundary condition at the point $(0,0)$. The method involves combining the computational solution of a classical finite difference operator on a tensor product of two piecewise-uniform Shishkin meshes with an analytical function that captures the local nature of the incompatibility in the problem data. A proof is given to show almost first order parameter-uniform pointwise global convergence of these numerical/analytical approximations.

Recovered element methods on polygonal meshes for convection-diffusion problems

Tristan Pryer *(University of Reading)* & E. Georgoulis and Z. Dong

We introduce a family of Galerkin finite element methods which are constructed via recovery operators over element-wise discontinuous approximation spaces. This new family, termed collectively as recovered finite element methods (R-FEM) has a number of attractive features over both classical finite element and discontinuous Galerkin approaches. A key one is the ability to produce conforming discretizations, involving only as many degrees of freedom as discontinuous Galerkin methods over general polygonal/polyhedral meshes with potentially many faces per element. This allows for the potential to produce stable conforming approximations in a variety of settings. Moreover, for special choices of recovery operators, R-FEM produces the same approximate solution as the classical conforming finite element method, while, trivially, one can recast (primal formulation) discontinuous Galerkin methods. In this talk we will present the method for general linear, possibly de-generate, second order advection-diffusion-reaction boundary value problems.

Sensitivity of mixing layers – why it is difficult to obtain reliable simulations for 2D Kelvin–Helmholtz instability problems

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Two-dimensional (2D) *mixing layers*, or *free shear layers*, are important phenomena in fluid dynamics. So-called *Kelvin–Helmholtz* (KH) instabilities arise as a result of impressing small perturbations on a shear layer [1]. Therefore, one observes merging of vortical structures in the considered motion. In the literature, such Kelvin–Helmholtz instability problems are frequently used to assess the quality of numerical methods for incompressible flow problems. However, it turns out that even if the same flow configuration is used, the numerical results usually differ significantly.

In this talk, based on [2], we present computational studies for a benchmark KH problem using a high-order exactly divergence-free $H(\mathrm{div})$-conforming HDG finite element method. Reliable results can be obtained as long as the simulation times are not too large. For large simulation times, perturbations accumulate which lead to different solutions. We identify some of those possible perturbations stemming from the numerical method used. All results are available online for the purpose of comparison [3].

In the case of high Reynolds number flows, it is well known that 2D flows behave completely different compared to the 3D situation – a phenomenon also connected to the notion of *2D turbulence* [4]. Here, *enstrophy* and *palinstrophy* play a crucial role. Beginning with a flow dominated by small-scale structures, a 2D flow tends to reorganise itself into larger structures (in 3D, it is exactly vice versa) [5]. In order to provide a possible explanation for the sensitivity of the KH instability problem, this theory is applied.

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Centred splash of a vertical jet on a horizontal rotating disc: the thin radial film in the parabolic and weakly elliptic limit

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Consider a liquid Newtonian jet impacting vertically, in direction of gravity, on the centre of a spinning, horizontally aligned, rigid, and perfectly smooth disc. We examine the so generated steady axisymmetric flow downstream of the nozzle issuing the jet, both analytically and numerically, in previously unattended detail and rigour [1]. Most emphasis is placed on the radial spread of the relatively thin free-surface layer forming adjacent

to the disc. Specifically, the Reynolds number Re, typical of the (contracted) jet just above impingement is assumed to be very large but sufficiently small to prevent laminar–turbulent transition at any position in the flow. The remaining principal assumptions are: the jet is very slender; the Froude number Fr and Weber number We characteristic of the thin-film regime are very large too. Notably, the herewith stipulated basic flow configuration applies to the vast amount of the familiar engineering applications.
Most important, the extreme geometrical conditions and the largeness of the non-dimensional key groups render the flow problem archetypal for the application of matched asymptotic expansions to cope rigorously with the involved scale separation. We first elucidate the predominantly inviscid jet and its deflection by the disc, described by an Euler problem that accounts for capillarity given the strongly varying curvature of the free surface, Σ, there. It is then demonstrated how the boundary layers first growing slowly along Σ and the disc surface beneath the impacting jet merge to eventually form the developed thin film. This evolves at a radial extent measured by $Re^{1/3}$ relative to the radius of the impinging jet. To leading order, it is governed by parabolic shallow-water equations. Their initial conditions impose an azimuthal vorticity, generated at jet formation by the specific shape of the nozzle and distorted through jet bending. Most important, in a least-degenerate limit a suitably formed Rossby number Ro, measuring the centrifugal fluid force, enters the equations as the sole control parameter. In a parametric study, particular emphasis is laid on the regular limit $Ro \to \infty$ (slowly rotating disc) and the singular one $Ro \to 0$ (strongly rotating disc). Varying also the upstream vorticity by virtue of the nozzle shape points to an intriguing control strategy of high practical relevance. For small values of Ro, incomplete similarity on a rescaled radial length scale $Ro^{-1/8}Re^{1/3}$ allows for a canonical representation of the flow. Here the classical complete similarity law found for a stationary disc [2] provides the upstream condition.
Also, we consider the global elliptic or upstream influence of finite values of Fr and/or We [4]. Impeding the formation of a fully developed state of the flow [3], they rather require the formulation of a downstream condition in terms of an expansive singularity prescribed at the disc edge [5].

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An adaptive HDG method for the Brinkman problem

Rodolfo Araya, Manuel Solano & **Patrick Vega** *(CI²MA and Department of Mathematical Engineering, Universidad de Concepción)*

We introduce and analyze a hybridizable discontinuous Galerkin (HDG) method for the gradient-velocity-pressure formulation of the Brinkman problem. We present an *a priori* error analysis of the method, showing optimal order of convergence of the error. We also introduce an *a posteriori* error estimator, of the residual type, which helps us to improve the quality of the numerical solution. We establish reliability and local efficiency of our estimator for the L^2-error of the velocity gradient and the pressure and the H^1-error of the velocity, with constants which are independent of the physical parameters and the size of the mesh. In particular, our results are also valid for the Stokes problem. Finally, we provide numerical experiments showing the quality of our adaptive scheme.

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Robust and efficient guaranteed error bounds for reaction-diffusion problems

Tomáš Vejchodský *(Institute of Mathematics, Czech Academy of Sciences)* & Mark Ainsworth *(Division of Applied Mathematics, Brown University)*

We propose a simple flux reconstruction and prove the robust efficiency of the resulting error bounds. We consider guaranteed error bounds for the finite element solution of the reaction-diffusion problem

$$-\Delta u + \kappa^2 u = f \quad \text{in } \Omega; \qquad u = 0 \quad \text{on } \partial\Omega.$$

These error bounds [1,2,3] are based on a flux reconstruction and the known complementarity technique [4]. We propose a simple flux reconstruction and prove local efficiency and robustness of the resulting error bounds. The robustness result implies efficient behaviour of the error bound even in the singularly perturbed limit $\kappa \to \infty$. The proposed flux reconstruction is inspired by [5] and it is based on small local problems on patches of elements solved by standard Raviart-Thomas finite elements. These local problems are independent and can be solved in parallel. As a result, the proposed error estimator is easy to implement, provides guaranteed upper bound on the energy norm of the total error, is locally efficient and robust with respect to any mesh size and any size of the reaction coefficient κ. These properties make it very attractive for practical computations.

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Numerics and perturbation analysis for some mixed type problems

Marcus Waurick *(University of Strathclyde)*

We discuss a numerical scheme for time-dependent partial differential equations that may change its type from parabolic to hyperbolic to elliptic type problems on different (rough) spatial domains. We shall further apply this scheme to a periodic 1+1-dimensional homogenisation type problem where the change of type is highly oscillatory. The results have applications to solid-fluid interaction models as well as to electromagnetic problems, where on some parts of the underlying domain an eddy current approximation is considered, whereas on other parts the full time-dependent Maxwell equations are treated.
This is joint work with Sebastian Franz and Sascha Trostorff, both from TU Dresden.

An *hp* finite element method for a reaction-convection-diffusion problem with two small parameters

Christos Xenophontos & I. Sykopetritou *(University of Cyprus)*

We consider a second order singularly perturbed boundary value problem, of reaction-convection-diffusion type with two small parameters, and the approximation of its solution by the *hp* version of the finite element method on the so-called *Spectral Boundary Layer* mesh. We show that the method converges uniformly, with respect to both singular perturbation parameters, at an exponential rate when the error is measured in the energy norm. Our theoretical findings are illustrated through numerical examples.

A stabilised finite element method for the convection-diffusion-reaction equation in mixed form

Heather Yorston *(University of Strathclyde)* & G.R. Barrenechea & A. Poza

The purpose of this talk is twofold. First, we present a new stabilised finite element method for a mixed formulation of the convection-diffusion equation. This method originated from a study aimed at facilitating the error analysis of Masud & Kwak's method [1], which was lacking in the original work. However, it developed into a separate new method, which is accompanied by error analysis, and which has subsequently been tested.

As a second step, we compare the new method to a variety of pre-existing mixed methods for the convection-diffusion equation with two different standard benchmarks in two space dimensions. Both qualitative and quantitative comparisons are presented. These numerical results confirm the error estimates and show that this present method appears as a competitive alternative to previously existing mixed methods for the CDR equation.

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